SOLUTIONS OF THE STANDARD DIFFERENTIAL EQUATION OF THE DYNAMIC GRAVITATIONAL FIELD OF A GALAXY

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In the tensor of energy-momentum of the relativistic gravithermodynamics (RGTD) not only intranuclear pressure p_N but also intranuclear temperature T_N is taken into account [1 – 3]:

$$b'_c / a_c b_c r - r^{-2}(1 - 1/a_c) + \Lambda = \kappa(T_N S_N - p_N V_N)/V = \kappa(m_{gr} - m_{in})c^2/V = \kappa m_{00} c^2 (1/\sqrt{b_c} - \sqrt{b_c})/V, \quad (1)$$

$$a'_c / a_c^2 r + r^{-2}(1 - 1/a_c) - \Lambda = \kappa E/V = \kappa m_{in} c^2 / V = \kappa m_{00} c^2 \sqrt{b_c}/V,$$

$$[\ln(b_c a_c)]' / a_c r = \kappa W / V = \kappa m_{gr} c^2 / V = \kappa m_{00} c^2 / \sqrt{b_c} V,$$

where: b_c and a_c are the parameters of the dynamic gravitational field equations of the non-continuous matter of the galaxy; $p_V V_N = \tilde{\beta}_{pVN} E = b_c \tilde{\beta}_{pVN} m_{gr} c^2 = \tilde{\beta}_{pVN} m_{in} c^2$, $\tilde{\beta}_{pVN} \neq \mathbf{const}(r)$, $S_N = m_{gr} c^2 / T_N = m_{00} c^2 / T_{00} = \mathbf{const}(r)$, $T_{00N} = T_N \sqrt{b_c} = \mathbf{const}(r)$, $m_{00} = m_{gr}\sqrt{b_c} = m_{in}/\sqrt{b_c} = \mathbf{const}(r)$, $\mu_{00} = m_{00}/V \neq \mathbf{const}(r)$, $\mu_{gr} = m_{00}/\sqrt{b_c} V = \mu_{in}/b_c \neq \mathbf{const}(r)$, $\mu_{in} = m_{00}\sqrt{b_c}/V \neq \mathbf{const}(r)$, $V \neq \mathbf{const}(r)$ and $V_N \neq \mathbf{const}(r)$ are molar and intranuclear volume of matter, respectively; W and E are the ordinary internal energy and inert free energy of matter, respectively.

In addition, according to the RGTD equations, the configuration of the dynamic gravitational field of a galaxy in a quasi-equilibrium state is standard (canonical in RGTD). Because it is not determined at all by the spatial distribution of the average mass density of its non-continuous matter. After all, this spatial distribution of the average mass density of the galaxy's matter is itself determined by the standard configuration of its dynamic gravitational field:

$$S' = \frac{d[r/a_c(1-b_c)]}{dr} = \frac{1 - r'_g - \Lambda r^2}{(1-b_c)} + \frac{(r - r_g - \Lambda r^3/3)}{(1-b_c)^2} b'_c = -\frac{b_c S}{r(1-b_c)} + \frac{(1-\Lambda r^2)}{(1-b_c)^2}, \quad (2)$$

$$S = \frac{r}{a_c(1-b_c)} = \frac{r - r_g - \Lambda r^2/3}{1-b_c} = \exp\int\frac{-b_c dr}{(1-b_c)r} \times \int\left[\frac{(1-\Lambda r^2)}{(1-b_c)^2}\exp\int\frac{b_c dr}{(1-b_c)r}\right]dr,$$

where the parameter S can be conditionally considered as the distance from the event pseudo-horizon.

The trivial solution of this equation, which takes place at:

$$b_c = b_{ce}\left(\frac{3-\Lambda r^2}{3-\Lambda r_e^2}\right), \quad S_0 = \frac{r - \Lambda r^3/3}{1-b_c} = \frac{(r-\Lambda r^3/3)(3-\Lambda r_e^2)}{3 - \Lambda r_e^2 - b_{ce}(3-\Lambda r^2)}, \quad r_g = \frac{(1-b_c)r_{ge}}{(1-b_{ce})}\exp\int_{r_{ge}}^{r_g}\frac{b_c dr}{r(1-b_c)} =$$

$$= \frac{(1-b_c)r_{ge}}{(1-b_{ce})}\exp\frac{2b_{ce}\ln(r/r_e) - (1-\Lambda r_e^2/3)\{\ln[r^2 + (3/\Lambda - r_e^2)/b_{ce} - 3/\Lambda] - \ln[(1/b_{ce}-1)(3/\Lambda - r_e^2)]\}}{2(1-\Lambda r_e^2/3 - b_{ce})},$$

does not correspond to physical reality. After all, because of $b'_c = -2b_{ce}\Lambda r/(3-\Lambda r_e^2) \neq 0$ at $r \neq 0$, the solution does not imply the presence of event pseudo-horizon in the frame of references of coordinates and time (FR) of matter. And the parameter b_c, unlike the parameter a_c, does not depend on the gravitational radius r_g. And therefore, gravity is absent in the FR corresponding to this trivial solution.

According to the non-identity of the gravitational and inert masses of matter we find the square of the rotation velocity of astronomical object relatively to the galaxy center according to the equations (2, 3) of gravitational field of RGTD:

$$\hat{v}^2 = \frac{c^2 r}{b_c}\frac{d\ln(v_{lc}/c)}{dr} = \frac{c^2 r b_c'}{2b_c^2} = \frac{c^2 a_c}{2b_c}\left\{(1-1/a_c) + \left[\kappa m_{00}c^2\left(1/\sqrt{b_c}-\sqrt{b_c}\right)/V - \Lambda\right]r^2\right\} >> [\hat{v}^2]_{GR} \quad (3)$$

As we can see, at the same radial distribution of the average density of the mass $\mu_{00} = m_{00}/V$ of baryonic matter the circular velocities of rotation of astronomical objects relatively to the galaxy center are much bigger in RGTD than in general relativity (GR). And this is, of course, related to the fact that:

$$(T_N S_N - p_N V_N)/V \equiv (m_{gr} - m_{in})c^2/V = \mu_{00}c^2(1/\sqrt{b_c} - \sqrt{b_c}) >> p\,.$$

Therefore, the fictitious need for dark non-baryonic matter in galaxies (which follows from the GR gravitational field equations) can be completely eliminated if the motion of astronomical objects is analyzed using the RGTD gravitational field equations and diffeomorphically-conjugated forms [4]:

$$\hat{v} = \frac{v}{\sqrt{b_c}} = \sqrt{\frac{2\,\mathrm{LH}_e(b_c/b_{ce})^n}{\mathrm{HL}_e[1+(b_c/b_{ce})^{2n}]}}\hat{v}_e = \sqrt{\frac{2(b_c/b_{ce})^n}{b_c[1+(b_c/b_{ce})^{2n}]}}v_e = \frac{v_{\max}}{\sqrt{b_c}}\left\{1+\frac{4n^2v_e^4}{c^4}\left[\ln\left(\frac{r-\Lambda r^3/3}{r_e-\Lambda r_e^3/3}\right) - u\ln\left(\frac{1-b_c}{1-b_{ce}}\right)\right]^2\right\}^{-1/4},$$

$$r - \frac{\Lambda r^3}{3} = \frac{(r_e-\Lambda r_e^3/3)(1-b_c)^u}{(1-b_{ce})^u}\exp\left[\pm\frac{c^2}{2n}\sqrt{v^{-4}-v_e^{-4}}\right] = \frac{(r_e-\Lambda r_e^3/3)(1-b_c)^u}{(1-b_{ce})^u}\exp\left\{\frac{c^2v_{\max}^{-2}}{4n}\left[\left(\frac{b_c}{b_{ce}}\right)^n - \left(\frac{b_{ce}}{b_c}\right)^n\right]\right\},$$

$$1/a_c = 1 - r_g/r - \Lambda r^2/3, \qquad b_c = k_b b_{ce} = b_{ce}\left[v_{\max}^2 v^{-2} \pm \sqrt{v_{\max}^4 v^{-4} - 1}\right]^{1/n} =$$

$$= b_{ce}\left\{\sqrt{1+\frac{4n^2v_e^4}{c^4}\left[\ln\left(\frac{r-\Lambda r^3/3}{r_e-\Lambda r_e^3/3}\right) - u(b_c)\ln\left(\frac{1-b_c}{1-b_{ce}}\right)\right]^2} \pm \frac{2nv_e^2}{c^2}\left[\ln\left(\frac{r-\Lambda r^3/3}{r_e-\Lambda r_e^3/3}\right) - u(b_c)\ln\left(\frac{1-b_c}{1-b_{ce}}\right)\right]\right\}^{1/n},$$

$$b_c' = \frac{db_c}{dr} = \frac{(1-\Lambda r^2)}{\left(r-\frac{\Lambda r^3}{3}\right)\left\{\frac{c^2}{2v_e^2 b_c}\sqrt{1+\frac{4n^2v_e^4}{c^4}\left[\ln\left(\frac{r-\Lambda r^3/3}{r_e-r_e^3/3}\right) - u(b_c)\ln\left(\frac{1-b_c}{1-b_{ce}}\right)\right]^2} - \frac{u(b_c)}{1-b_c} + \ln(1-b_c)\frac{du}{db_c}\right\}} =$$

$$= \frac{(1-\Lambda r^2)}{\left(r-\frac{\Lambda r^3}{3}\right)\left\{\frac{c^2}{4v_e^2 b_c}\left[\left(\frac{b_c}{b_{ce}}\right)^n + \left(\frac{b_{ce}}{b_c}\right)^n\right] - \frac{u(b_c)}{1-b_c} + \ln\left(\frac{1-b_c}{1-b_{ce}}\right)\frac{du}{db_c}\right\}}, \quad \frac{b_c'}{b_c a_c r} - \frac{1}{r^2}\left(1-\frac{1}{a_c}\right) + \Lambda - \frac{\kappa m_{00}c^2}{V}\left(\frac{1}{\sqrt{b_c}} - \sqrt{b_c}\right) =$$

$$= \frac{(1-\Lambda r^2)(r^{-2} - r_g r^{-3} - \Lambda/3)}{\left(1-\frac{\Lambda r^2}{3}\right)\left\{\frac{c^2}{4v_e^2}\left[\left(\frac{b_c}{b_{ce}}\right)^n + \left(\frac{b_{ce}}{b_c}\right)^n\right] - b_c\left[\frac{u(b_c)}{1-b_c} - \ln\left(\frac{1-b_c}{1-b_{ce}}\right)\frac{du}{db_c}\right]\right\}} - \frac{r_g}{r^3} + \frac{2\Lambda}{3} - \frac{\kappa m_{00}c^2}{V}\left(\frac{1}{\sqrt{b_c}} - \sqrt{b_c}\right) = 0,$$

$$V = \frac{\kappa m_{00}c^2(1-\Lambda r^2/3)\left\{(1/\sqrt{b_{ce}})\left[\sqrt{1+A^2} \mp A\right]^{\frac{1}{2n}} - \sqrt{b_{ce}}\left[\sqrt{1+A^2} \pm A\right]^{\frac{1}{2n}}\right\}\left(\sqrt{1+A^2} - B\right)}{2v_e^2c^{-2}(1-\Lambda r^2)(r^{-2} - r_g r^{-3} - \Lambda/3) - (1-\Lambda r^2/3)(r_g r^{-3} - 2\Lambda/3)\left(\sqrt{1+A^2} - B\right)},$$

$$\mu_{gr} = \frac{m_{00}}{\sqrt{b_c}V} = \frac{2v_e^2(1-\Lambda r^2)(r^{-2} - r_g r^{-3} - \Lambda/3)}{\kappa c^4(1-b_c)(1-\Lambda r^2/3)\left(\sqrt{1+A^2} - B\right)} + \frac{2\Lambda/3 - r_g r^{-3}}{\kappa c^2(1-b_c)}, \quad \mu_{gr\min} = \frac{2\Lambda/3}{\kappa c^2(1-b_{c\max})} = \frac{2H_E^2}{\kappa c^4(1-b_{c\max})},$$

where: $A = \frac{2nv_e^2}{c^2}\left[\ln\left(\frac{r-\Lambda r^3/3}{r_e - \Lambda r_e^3/3}\right) - u(b_c)\ln\left(\frac{1-b_c}{1-b_{ce}}\right)\right]$, $B = \frac{2b_c v_e^2}{c^2}\left[\frac{u(b_c)}{1-b_c} - \ln\left(\frac{1-b_c}{1-b_{ce}}\right)\frac{du}{db_c}\right]$, $r_g = r_{ge} + \int_{r_e}^{r} r_g' dr$,

r_e is radius of the conventional friable galactic nucleus, on the surface of which the corrected value $\hat{v}$ of the orbital velocity of objects can take its maximum possible value $v_{\max} \equiv v_e = b_{ce}^{1/2}\hat{v}_e(b_e) = v_{lce}\hat{v}_e / c$; r_g and r_{ge} are the gravitational radii of any layer of the galaxy and its loose core, respectively.

The dependence of the gravitational radii of a galaxy on the radial coordinate is determined from the following differential equation:

$$r_g' = \kappa\mu_{in}c^2 r^2 = \frac{\frac{2v_e^2(1-\Lambda r^2)}{c^2(1-\Lambda r^2/3)\left(\sqrt{1+A^2} - B\right)}\left(1 - \frac{r_g}{r} - \frac{\Lambda r^2}{3}\right) - \left(\frac{r_g}{r} - \frac{2\Lambda r^2}{3}\right)}{\frac{1}{b_{ce}}\left\{\sqrt{1 + \frac{4n^2v_e^2}{c^2}\left[\ln\left(\frac{r-\Lambda r^3/3}{r_e - \Lambda r_e^3/3}\right) - u(b_c)\ln\left(\frac{1-b_c}{1-b_{ce}}\right)\right]^2} \mp \frac{2nv_e^2}{c^2}\left[\ln\left(\frac{r-\Lambda r^3/3}{r_e - \Lambda r_e^3/3}\right) - u(b_c)\ln\left(\frac{1-b_c}{1-b_{ce}}\right)\right]\right\}^{\frac{1}{n}} - 1},$$

or using dependent on it parameter S:

$$dS = d\left(\frac{r - r_g - \Lambda r^3/3}{1-b_c}\right) = -\left\{\frac{c^2}{4v_e^2 b_c}\left[\left(\frac{b_c}{b_{ce}}\right)^n + \left(\frac{b_{ce}}{b_c}\right)^n\right] - \frac{u(b_c)}{1-b_c} + \ln\left(\frac{1-b_c}{1-b_{ce}}\right)\frac{du}{db_c}\right\}\left(1 - \frac{\Lambda r^2}{3}\right)\left[\frac{b_c S}{(1-\Lambda r^2)(1-b_c)} - \frac{r}{(1-b_c)^2}\right]db_c,$$

$$r_g = r - \frac{\Lambda r^3}{3} - (1-b_c)\exp\left[-\int\frac{b_c dr}{(1-b_c)r}\right] \times \int\left\{\frac{1-\Lambda r^2}{(1-b_c)^2}\exp\left[\int\frac{b_c dr}{(1-b_c)r}\right]\right\}dr = r - \frac{\Lambda r^3}{3} -$$

$$-\frac{c^2(r_e - \Lambda r_e^3/3)(1-b_c)}{4v_e^2}\exp\left[-\int\frac{b_c dr}{(1-b_c)r}\right] \times \int_{b_{ce}}^{b_c}\left\{\frac{[(b_c/b_{ce})^n + (b_{ce}/b_c)^n]}{b_c(1-b_c)^2} - \frac{4v_e^2c^{-2}u}{(1-b_c)^3}\right\}\exp\left\{\frac{c^2}{4nv_e^2}\left[\left(\frac{b_c}{b_{ce}}\right)^n - \left(\frac{b_{ce}}{b_c}\right)^n\right] + \int\frac{b_c dr}{(1-b_c)r}\right\}db_c =$$

$$= \frac{c^2(r_e - \Lambda r_e^3/3)(1-b_c)}{4v_e^2}\exp\left[-\int\frac{b_c dr}{(1-b_c)r}\right] \times \int_{b_{ce}}^{b_c}\left\{\left[1 - \ln\left(\frac{1-b_c}{1-b_{ce}}\right)\left(\frac{b_c(1-\Lambda r^2/3)}{1-\Lambda r^2} - 1\right)\frac{du}{db_c}\right]\frac{1}{(1-b_c)^2} - \right.$$

$$\left. - \frac{u}{(1-b_c)^3}\left[\frac{b_c(1-\Lambda r^2/3)}{1-\Lambda r^2} - 1\right] + \frac{\Lambda c^2[(b_c/b_{ce})^n + (b_{ce}/b_c)^n]}{6v_e^2(r^{-2} - \Lambda)(1-b_c)^2}\right\}\exp\left\{\frac{c^2}{4nv_e^2}\left[\left(\frac{b_c}{b_{ce}}\right)^n - \left(\frac{b_{ce}}{b_c}\right)^n\right] + \int\frac{b_c dr}{(1-b_c)r}\right\}db_c,$$

where: $\int\frac{b_c dr}{(1-b_c)r} = \int\frac{1-\Lambda r^2/3}{(1-\Lambda r^2)(1-b_c)}\left\{\frac{c^2}{4v_e^2}\left[\left(\frac{b_c}{b_{ce}}\right)^n + \left(\frac{b_{ce}}{b_c}\right)^n\right] - \frac{b_c u}{1-b_c} + b_c\ln\left(\frac{1-b_c}{1-b_{ce}}\right)\frac{du}{db_c}\right\}db_c$.

At u=-1 ($v_e = c/\sqrt{2}$) this solution of the standard equation of the dynamic gravitational field of the galaxy allegedly degenerates. After all, in this case the value of the gravitational radius of the galaxy becomes proportional to the cosmological constant Λ, and therefore to the Hubble constant:

$$r_g = \frac{2\Lambda(3r_e - \Lambda r_e^3)(1-b_c)}{9}\exp\left[-\int\frac{b_c dr}{(1-b_c)r}\right] \times \int_{b_{ce}}^{b_c}\frac{r^2\{b_c + c^2v_e^{-2}(1-b_c)[(b_c/b_{ce})^n + (b_{ce}/b_c)^n]/4\}}{(1-\Lambda r^2)(1-b_c)^3}\exp\left\{\frac{c^2}{4nv_e^2}\left[\left(\frac{b_c}{b_{ce}}\right)^n - \left(\frac{b_{ce}}{b_c}\right)^n\right] + \int\frac{b_c dr}{(1-b_c)r}\right\}db_c.$$

But in fact, like the parameter b_c, the cosmological constant is a hidden parameter of matter. And it is thanks to it that at $b_{ce} > (1-\Lambda r_e^2)/(1-\Lambda r_e^2/3)$ and at $u = -c^2v^{-2}/2$ the radial gravitational radii $r_g(r)$ of the galaxy become larger than at u=0.

Also what is important is that even in an incredibly weak gravitational field (when u=0) and even at large radial distances, astronomical objects will rotate around the center of the galaxy with orbital velocities very close to the maximum possible speed [5, 6]. After all, the radial distances to the objects of the galaxy at the same value of the parameter b_c become much greater than at u=0:

$$\frac{dr}{db_c} = \frac{c^2(r-\Lambda r^3/3)}{4v_e^2 b_c(1-\Lambda r^2)}\left\{\frac{1}{1-b_c}\left[\left(\frac{b_c}{b_{ce}}\right)^n + \left(\frac{b_{ce}}{b_c}\right)^n\right] - n\ln(1-b_c)\left[\left(\frac{b_c}{b_{ce}}\right)^n - \left(\frac{b_{ce}}{b_c}\right)^n\right]\right\} >> \frac{c^2(r-\Lambda r^3/3)}{4v_e^2 b_c(1-\Lambda r^2)}\left[\left(\frac{b_c}{b_{ce}}\right)^n + \left(\frac{b_{ce}}{b_c}\right)^n\right].$$

References

1. Danylchenko, Pavlo: 2021, Solution of equations of the galaxy gravitational field. *Proceed. Fourth Int. Conference APFS'2021*. Lutsk: Volyn University Press "Vezha" [ISBN: 978-966-940-362-9], 33-36, https://elibrary.com.ua/m/articles/view/SOLUTION-OF-EQUATIONS-OF-THE-GALAXY-GRAVITATIONAL-FIELD-2021-05-27.
2. Danylchenko, Pavlo: 2022, *Foundations of Relativistic Gravithermodynamics*. Vinnytsia: TVORY [ISBN: 978-617-552-072-7] (in Ukrainian).
3. Danylchenko, Pavlo: 2024, *Foundations of Relativistic Gravithermodynamics 5th online edition*, https://elibrary.com.ua/m/articles/view/FOUNDATIONS-OF-RELATIVISTIC-GRAVITHERMODYNAMICS-5th.
4. Trokhimchuck, Petro P.: 1985, Contradictions in modern physical theory. The method of diffeomorphically-conjugated forms and some its applications, Preprint USC AS the USSR, Sverdlovsk.
5. Pogge, Richard: Lecture 41: 2006. Dark Matter & Dark Energy. *Astronomy 162: Introduction to Stars, Galaxies, & the Universe*, http://www.astronomy.ohio-state.edu/~pogge/Ast162/Unit6/dark.html.
6. Bennett, Jeffrey, Donahue Megan, et al.: 2012, The essential cosmic perspective. Boston: Addison-Wesley, The 8th Edition 2017.