RELATIVISTIC TRANSFORMATIONS OF COORDINATE INCREMENTS AND METRIC SEGMENTS OF BODIES MOVING IN A GRAVITATIONAL FIELD BY INERTIA

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Increments of coordinate time and spatial coordinates can be expressed not only in four-dimensional pseudo-Euclidean Minkowsky space, but also in similar to it four-dimensional hyperbolic space [1]. Let us examine at first the movement along the one direction. According to Lorentz transformations we will have:

$$cdt' = \mathrm{ch}(\psi_t - \psi_{t0})ds = \frac{1 - vv_0c^{-2}}{\sqrt{(1-v^2c^{-2})(1-v_0^2c^{-2})}}ds = \frac{\mathrm{ch}(\psi_t - \psi_{t0})}{\mathrm{ch}\psi_t}cdt = \sqrt{\frac{1-v^2c^{-2}}{1-v'^2c^{-2}}}cdt\,,$$

$$dl' = \mathrm{sh}(\psi_t - \psi_{t0})ds = \frac{(v-v_0)/c}{\sqrt{(1-v^2c^{-2})(1-v_0^2c^{-2})}}ds = \frac{\mathrm{sh}(\psi_t - \psi_{t0})}{\mathrm{sh}\psi_t}dl = \frac{v'}{v}\sqrt{\frac{1-v^2c^{-2}}{1-v'^2c^{-2}}}dl\,,$$

$$v' = \frac{dl'}{dt'} = c\ \mathrm{th}(\psi_t - \psi_{t0}) = \frac{v-v_0}{1-vv_0c^{-2}} = \frac{\mathrm{th}(\psi_t - \psi_{t0})}{\mathrm{th}\psi_t}v\,,$$

where: $ds = cd\hat{t} = \sqrt{1-v'^2c^{-2}}\,dt' = \sqrt{1-v^2c^{-2}}\,dt = \mathbf{inv}$, $\mathrm{th}\psi_t = v/c$, $\mathrm{sh}\psi_t = (v/c)(1-v^2c^{-2})^{-1/2}$, $\mathrm{ch}\psi_t = (1-v^2c^{-2})^{-1/2}$.

The considerations that instead of relativistic shrinkage of longitudinal size of moving body the increasing of it can take place were expressed by various authors [2 - 4]. However, it can take place only for forcibly accelerated bodies. Anyway the shrinkage of the size in background Euclidean space of the observer should take place for inertially moving bodies. Due to the necessity to fulfill the general covariance of the physical equations the change of metrical size of moving body in intrinsic space (that has curvature) of the observer should, of course, be absent [5, 6].

And this means that Lorentz transformations are the transformations of only spatial coordinates and coordinate time and not of metrical spatial segments or metrical time intervals. And, therefore, it is still required to multiply the matrix of transformations of increments of coordinates [1] by the matrix of transition to the increments of metrical segments. This matrix should be similar to the matrix of metrical tensor of GR and, of course, it should include not only the indexes of local curvature of time (values of local coordinate velocities of light $v_{cv} = c(dt/d\hat{t}) = v_{cvr}(1-v^2v_{cv}^{-2})^{1/2}$), but also direct or reverse indexes of local curvature of the space:

$$\xi = d\hat{x}/dx = 1/\zeta = c/v_{cv} = \xi_r(1-v^2v_{cv}^{-2})^{-1/2}\,, \qquad \zeta = dx/d\hat{x} = v_{cv}/c = \zeta_r(1-v^2v_{cv}^{-2})^{1/2}\,,$$

where: dt and $d\hat{t}$ are the increments of the spatially inhomogeneous coordinate (gravity-quantum) time in background pseudo-Euclidean STC and of the united intrinsic (gravithermodynamic) time of all gravithermodynamically bonded matter correspondingly; dx and $d\hat{x}$ are the increments of spatial segments in background Euclidean space and in intrinsic space (that has local curvature) of matter correspondingly; $v_{cvr} = c\zeta_r = c/\xi_r$ and ξ_r, ζ_r are correspondingly the coordinate velocity of light and also the direct and reverse values of indexes of curvature of the space in the hypothetic point FR_{out} of outer observer, in which moving body could be in the state of rest. At the same time, in the same way as in conventionally empty space in GR, $v_{cv}\xi = v_{cvr}/\zeta_r = c$.

However, these indexes that take different values in different FRs can be as well directly included in the modernized Lorentz transformations themselves.

So, according to increment of metrical segment $d\widehat{x} = \xi dx = \xi_r(1 - v^2 v_{cv}^{-2})^{-1/2} dx$ the real metrical velocity of inertial motion of the body in the uniform gravithermodynamic time (world time of GR [1]) of FR_{out} will be de facto equal to its intrinsic value in the FR of moving body [7, 8]:

$$\widehat{v} = d\widehat{x}/d\widehat{t} = (\xi v_{cv}/c) dx/dt = dx/dt = vc/v_{cv} = (vc/v_{cvr})(1 - v^2 v_{cv}^{-2})^{-1/2} = (vc/v_{cvr})\left[\left(1 + \sqrt{1 - 4v^2 v_{cvr}^{-2}}\right)/2\right]^{-1/2} = c\sqrt{1 - v_{cv}^2 v_{cvr}^{-2}},$$

where: $\xi v_{cv} = c$; $v = dx/d\widehat{t} = \zeta\widehat{v} = \widehat{v}/\xi = \widehat{v} v_{cv}/c = \widehat{v}(v_{cvr}/c)(1 - \widehat{v}^2 c^{-2})^{1/2}$ is the velocity of motion of object in background regular space of FR_{out}, which does not take into account the curvature contributed by nearby astronomical objects and by the moving body itself.

And this means that not only the decreasing of coordinate velocity of light, but also the increasing of curvature of intrinsic space of FR_{out} is locally compensated by the inertial motion. It is obvious that gravitational curvature of regular space (gravitational decreasing of longitudinal size of the body that falls free in background Euclidean space) is completely compensated by the local curvature of this space, which appears due to the increasing of the velocity of inertial motion of body in gravitational field. And, consequently, the local coordinate velocity of light as well as indexes of local curvature of the space in the point of instantaneous dislocation of inertially moving body correspond in its intrinsic FR to their values ξ_r, ζ_r in hypothetical point of the start of independent motion of the body and not to their regular values in the points of instantaneous dislocation of the body in FR_{out}.

According to this, the transformations of the velocities projections of motion will have the following form [9]:

$$\frac{\widehat{v}'_{mx}}{c} = \frac{1}{c}\frac{dx'_m}{dt'} = \frac{\zeta' d\widehat{x}'_m}{v'_l d\widehat{t}'} = \frac{v'_{mx}}{v'_l} = \frac{v_{mx}/v_l - v_0/v_{l0}}{1 - v_0 v_{mx}/v_{l0} v_l} = \frac{\widehat{v}_{mx} - \widehat{v}_0}{c - \widehat{v}_0 \widehat{v}_{mx}/c},$$

$$\frac{\widehat{v}'_{my}}{c} = \frac{1}{c}\frac{dy'_m}{dt'} = \frac{\zeta' d\widehat{y}'_m}{v'_l d\widehat{t}'} = \frac{v'_{my}}{v'_l} = \frac{v_{my}}{v_l}\frac{\sqrt{1 - v_0^2 v_{l0}^{-2}}}{1 - v_0 v_{mx}/v_{l0} v_l} = \widehat{v}_{my}\frac{\sqrt{1 - \widehat{v}_0^2 c^{-2}}}{c - \widehat{v}_0 \widehat{v}_{mx}/c},$$

$$\frac{\widehat{v}'_{mz}}{c} = \frac{1}{c}\frac{dz'_m}{dt'} = \frac{\zeta' d\widehat{z}'_m}{v'_l d\widehat{t}'} = \frac{v'_{mz}}{v'_l} = \frac{v_{mz}}{v_l}\frac{\sqrt{1 - v_0^2 v_{l0}^{-2}}}{1 - v_0 v_{mx}/v_{l0} v_l} = \widehat{v}_{mz}\frac{\sqrt{1 - \widehat{v}_0^2 c^{-2}}}{c - \widehat{v}_0 \widehat{v}_{mx}/c}, \qquad \sqrt{1 - \widehat{v}'^2 c^{-2}} = \frac{\sqrt{(1 - \widehat{v}_0^2 c^{-2})(1 - \widehat{v}^2 c^{-2})}}{1 - \widehat{v}_0 \widehat{v}_{mx} c^{-2}},$$

$$\frac{\widehat{v}_{mx}}{c} = \frac{1}{c}\frac{dx_m}{dt} = \frac{\zeta d\widehat{x}_m}{v_l d\widehat{t}} = \frac{v_{mx}}{v_l} = \frac{v'_{mx}/v'_l - v'_0/v'_{l0}}{1 - v'_0 v'_{mx}/v'_{l0} v'_l} = \frac{\widehat{v}'_{mx} - \widehat{v}'_0}{c - \widehat{v}'_0 \widehat{v}'_{mx}/c}, \quad \frac{\widehat{v}_{my}}{c} = \frac{1}{c}\frac{dy_m}{dt} = \frac{\zeta d\widehat{y}_m}{v_l d\widehat{t}} = \frac{v_{my}}{v_l} = \frac{v'_{my}}{v'_l}\frac{\sqrt{1 - v'^2_0 v'^{-2}_{l0}}}{1 - v'_0 v'_{mx}/v'_{l0} v'_l} = \widehat{v}'_{my}\frac{\sqrt{1 - \widehat{v}'^2_0 c^{-2}}}{c - \widehat{v}'_0 \widehat{v}'_{mx}/c},$$

$$\frac{\widehat{v}_{mz}}{c} = \frac{1}{c}\frac{dz_m}{dt} = \frac{\zeta d\widehat{z}_m}{v_l d\widehat{t}} = \frac{v_{mz}}{v_l} = \frac{v'_{mz}}{v'_l}\frac{\sqrt{1 - v'^2_0 v'^{-2}_{l0}}}{1 - v'_0 v'_{mx}/v'_{l0} v'_l} = \widehat{v}'_{mz}\frac{\sqrt{1 - \widehat{v}'^2_0 c^{-2}}}{c - \widehat{v}'_0 \widehat{v}'_{mx}/c}, \qquad \sqrt{1 - \widehat{v}^2 c^{-2}} = \frac{\sqrt{(1 - \widehat{v}'^2_0 c^{-2})(1 - \widehat{v}'^2 c^{-2})}}{1 - \widehat{v}'_0 \widehat{v}'_{mx} c^{-2}},$$

where the real velocities of motion of observed object and of FR_0 are equal to $\widehat{v}_{mx} = v_{mx} c/v_l$, $\widehat{v}'_{mx} = v'_{mx} c/v'_l$, $\widehat{v}_{my} = v_{my} c/v_l$, $\widehat{v}'_{my} = v'_{my} c/v'_l$, $\widehat{v}_{mz} = v_{mz} c/v_l$, $\widehat{v}'_{mz} = v'_{mz} c/v'_l$ and $\widehat{v}_0 = v_0 c/v_{l0}$ correspondingly.

Thus, the inertial motion of the matter in gravitational field not only prevents the gravitational increasing of its refractive index of radiation, but also causes in the regular space of the CO_{out} the relativistic (kinematic) self-contraction of it in both longitudinal and transversal directions [10]. According to this the rate of metrical time of inertially moving bodies is unchangeable and relativistically invariant ($\zeta'_G = \zeta_G = \zeta_r = v_{lG}/c = v_{lr}/c$, $d\hat{t}' = d\hat{t}$). According to this we will have the following expressions for the transformation of increments of metrical segments ($d\hat{x}_m$, $d\hat{y}_m$, $d\hat{z}_m$) and coordinates (dt , dx_m , dy_m , dz_m):

$$d\hat{x}'_m = \frac{\zeta_G d\hat{x}_m - (\hat{v}_0/c)v_{lG}d\hat{t}}{\zeta'_G(1-\hat{v}_x\hat{v}_0c^{-2})} = \frac{\hat{v}_x - \hat{v}_0}{1-\hat{v}_x\hat{v}_0c^{-2}}d\hat{t} = \mathrm{th}(\psi_t - \psi_{t0})cd\hat{t} = \hat{v}'_x d\hat{t}' ,$$

$$d\hat{x}_m = \frac{\zeta'_G d\hat{x}'_m - (\hat{v}'_0/c)v'_{lG}d\hat{t}'}{\zeta_G(1-\hat{v}'_x\hat{v}'_0c^{-2})} = \frac{\hat{v}'_x - \hat{v}'_0}{1-\hat{v}'_x\hat{v}'_0c^{-2}}d\hat{t}' = \mathrm{th}(\psi'_t - \psi'_{t0})cd\hat{t}' = \hat{v}_x d\hat{t} ,$$

$$d\hat{y}'_m = \frac{d\hat{y}_m\sqrt{1-\hat{v}_0^2c^{-2}}}{1-\hat{v}_x\hat{v}_0c^{-2}} = \frac{\hat{v}_y\sqrt{1-\hat{v}_0^2c^{-2}}}{1-\hat{v}_x\hat{v}_0c^{-2}}d\hat{t} = \frac{\mathrm{ch}\psi_t}{\mathrm{ch}(\psi_t - \psi_{t0})}\hat{v}_y d\hat{t} = \hat{v}'_y d\hat{t}' ,$$

$$d\hat{y}_m = \frac{\mathrm{ch}(\psi_t - \psi_{t0})}{\mathrm{ch}\psi_t}d\hat{y}'_m = \frac{d\hat{y}'_m\sqrt{1-\hat{v}_0'^2c^{-2}}}{1-\hat{v}'_x\hat{v}'_0c^{-2}} = \frac{\hat{v}'_y\sqrt{1-\hat{v}_0'^2c^{-2}}}{1-\hat{v}'_x\hat{v}'_0c^{-2}}d\hat{t}' = \hat{v}_y d\hat{t} ,$$

$$d\hat{z}'_m = \frac{\mathrm{ch}\psi_t d\hat{z}_m}{\mathrm{ch}(\psi_t - \psi_{t0})} = \frac{d\hat{z}_m\sqrt{1-\hat{v}_0^2c^{-2}}}{1-\hat{v}_x\hat{v}_0c^{-2}} = \frac{\hat{v}_z\sqrt{1-\hat{v}_0^2c^{-2}}}{1-\hat{v}_x\hat{v}_0c^{-2}}d\hat{t} = \hat{v}'_z d\hat{t}' ,$$

$$d\hat{z}_m = \frac{d\hat{z}'_m\sqrt{1-\hat{v}_0'^2c^{-2}}}{1-\hat{v}'_x\hat{v}'_0c^{-2}} = \frac{\hat{v}'_z\sqrt{1-\hat{v}_0'^2c^{-2}}}{1-\hat{v}'_x\hat{v}'_0c^{-2}}d\hat{t}' = \frac{\mathrm{ch}\psi'_t}{\mathrm{ch}\psi_t}\hat{v}'_z d\hat{t}' = \hat{v}_z d\hat{t} ;$$

$$dt' = \frac{\sqrt{1-\hat{v}_0^2c^{-2}}(dt - \hat{v}_0c^{-2}dx_m)}{(1-\hat{v}_x\hat{v}_0c^{-2})^2} = \frac{\sqrt{1-\hat{v}_0^2c^{-2}}}{1-\hat{v}_x\hat{v}_0c^{-2}}dt = \sqrt{\frac{1-\hat{v}'^2c^{-2}}{1-\hat{v}^2c^{-2}}}dt = \frac{\mathrm{ch}\psi_t}{\mathrm{ch}(\psi_t - \psi_{t0})}dt \qquad (dx_m = \hat{v}_x dt),$$

$$dt = \frac{\sqrt{1-\hat{v}_0'^2c^{-2}}(dt' - \hat{v}'_0c^{-2}dx'_m)}{(1-\hat{v}'_x\hat{v}'_0c^{-2})^2} = \frac{\sqrt{1-\hat{v}_{0c}'^2c^{-2}}}{1-\hat{v}'\hat{v}'_0c^{-2}}dt' = \sqrt{\frac{1-\hat{v}^2c^{-2}}{1-\hat{v}'^2c^{-2}}}dt' = \frac{\mathrm{ch}\psi'_t}{\mathrm{ch}\psi_t}dt' \qquad (dx'_m = \hat{v}'_x dt),$$

$$dx'_m = \frac{\sqrt{1-\hat{v}_0^2c^{-2}}(dx_m - \hat{v}_0 dt)}{(1-\hat{v}_x\hat{v}_0c^{-2})^2} = \frac{\sqrt{1-\hat{v}_0^2c^{-2}}}{1-\hat{v}_x\hat{v}_0c^{-2}}dx_{ij} = \sqrt{\frac{1-\hat{v}'^2c^{-2}}{1-\hat{v}^2c^{-2}}}dx_{ij} = \frac{\mathrm{ch}\psi_t}{\mathrm{ch}(\psi_t - \psi_{t0})}dx_{ij} \qquad (dx_m = dx_{ij},\ dt = \hat{v}_x c^{-2}dx_{ij}),$$

$$dx_m = \frac{\sqrt{1-\hat{v}_0'^2c^{-2}}(dx'_m - \hat{v}'_0 dt')}{(1-\hat{v}'_x\hat{v}'_0c^{-2})^2} = \frac{\sqrt{1-\hat{v}_0'^2c^{-2}}}{1-\hat{v}'_x\hat{v}'_0c^{-2}}dx'_{ij} = \sqrt{\frac{1-\hat{v}^2c^{-2}}{1-\hat{v}'^2c^{-2}}}dx'_{ij} = \frac{\mathrm{ch}(\psi_t - \psi_{t0})}{\mathrm{ch}\psi_t}dx'_{ij} \qquad (dx'_m = dx'_{ij},\ dt' = \hat{v}'_x c^{-2}dx'_{ij}),$$

$$dy'_m = \frac{\mathrm{ch}^2\psi_t}{\mathrm{ch}^2(\psi_t - \psi_{t0})}dy_m = \frac{(1-\hat{v}_0^2c^{-2})}{(1-\hat{v}_x\hat{v}_0c^{-2})^2}dy_m , \qquad dy_m = \frac{1-\hat{v}^2c^{-2}}{1-\hat{v}'^2c^{-2}}dy'_m = \frac{(1-\hat{v}_0'^2c^{-2})}{(1-\hat{v}'_x\hat{v}'_0c^{-2})^2}dy'_m ,$$

$$dz'_m = \frac{1-\hat{v}'^2c^{-2}}{1-\hat{v}^2c^{-2}}dz_m = \frac{(1-\hat{v}_0^2c^{-2})}{(1-\hat{v}_x\hat{v}_0c^{-2})^2}dz_m , \qquad dz_m = \frac{\mathrm{ch}^2(\psi_t - \psi_{t0})}{\mathrm{ch}^2\psi_t}dz'_m = \frac{(1-\hat{v}_0'^2c^{-2})}{(1-\hat{v}'_x\hat{v}'_0c^{-2})^2}dz'_m ,$$

where: dx_{ij} and dx'_{ij} are the increments of coordinates that correspond to spatial transition to another object *j*, which is located in the same collective spatial-temporal microstate that the initial object *i* (while the increments dx_m and dx'_m correspond to the change in time of the spatial location of the same object *m*).

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